MATH 105 EXAM #2 (2.3-2.6) NAME_____

1. (8 points) Perform the following multiplication of complex numbers and write your final answer in standard form $a + bi$. Show all work.

$(6 + 5i)(4 - 3i)$

$= 24 - 18i + 20i - 15i^2$

$= 24 + 2i - 15(-1)$

$= 39 + 2i$

2. (8 points) Simplify the following expression as much as possible and write your final answer in standard form $a + bi$. Show all work.

$(7 + \sqrt{-12}) - (2 - \sqrt{-48})$

$= 7 + \sqrt{-4 \cdot 3} - 2 + \sqrt{-16 \cdot 3}$

$= 7 + 2\sqrt{3}i - 2 + 4\sqrt{3}i$

$= 5 + 6\sqrt{3}i$

3. (8 points) Perform the following division of complex numbers and write your final answer in standard form $a + bi$. Show all work.

$\frac{4 + 6i}{2 - i}$

$= \frac{(4 + 6i)}{(2 - i)} \cdot \frac{(2 + i)}{(2 + i)}$

$= \frac{8 + 4i + 12i + 6i^2}{2^2 + 1^2}$

$= \frac{8 + 16i - 6}{5}$

$= \frac{2 + 16i}{5}$

$= \frac{2}{5} + \frac{16}{5}i$

4. (8 points) Solve the following equation by extracting roots. Show all work.

$(2x + 1)^2 - 49 = 0$

$(2x + 1)^2 = 49$

$\sqrt{(2x + 1)^2} = \pm\sqrt{49}$

$2x + 1 = \pm 7$

$2x + 1 = 7$ $\qquad$ $2x + 1 = -7$

$2x = 6$ $\qquad$ $2x = -8$

$x = 3$ $\qquad$ $x = -4$

5. (8 points) Solve the following equation by factoring. Show all work.

$2x^2 - 19x - 33 = 0$

$(2x+3)(x-11) = 0$

$2x + 3 = 0$ $\quad$ $x - 11 = 0$

$2x = -3$ $\quad$ $x = 11$

$x = -\frac{3}{2}$

6. (9 points) Find all solutions (both real and complex) to the following equation. Show all work.

$x^4 - 5x^2 - 36 = 0$

$(x^2)^2 - 5(x^2) - 36 = 0$

Let $u = x^2$

$u^2 - 5u - 36 = 0$

$(u+4)(u-9) = 0$

$u + 4 = 0$ $\quad$ $u - 9 = 0$

$x^2 + 4 = 0$ $\quad$ $x^2 - 9 = 0$

$x^2 = -4$ $\quad$ $x^2 = 9$

$x = \pm\sqrt{-4}$ $\quad$ $x = \pm\sqrt{9}$

$x = \pm 2i$ $\quad$ $x = \pm 3$

7. (9 points) Find all solutions to the following equation. Show all work.

$\sqrt{5-x} - 1 = x$

$\sqrt{5-x} = x + 1$

$(\sqrt{5-x})^2 = (x+1)^2$

$5 - x = x^2 + 2x + 1$

$0 = x^2 + 3x - 4$

$(x+4)(x-1) = 0$

$x + 4 = 0$, $x - 1 = 0$

$x = -4$, $x = 1$

Check $x = -4$

$\sqrt{5-(-4)} - 1 \stackrel{?}{=} -4$

$\sqrt{9} - 1 \stackrel{?}{=} -4$

$2 \stackrel{?}{=} -4$

No. So -4 is not a solution

Check $x = 1$

$\sqrt{5-1} - 1 \stackrel{?}{=} 1$

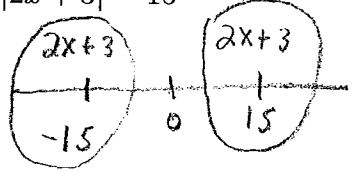
$\sqrt{4} - 1 \stackrel{?}{=} 1$

$2 - 1 \stackrel{?}{=} 1$

Yes. So $x = 1$ is the only solution.

8. (8 points) Find all solutions to the following equation. Show all work.

$|2x + 3| = 15$



$2x + 3 = -15$ OR $2x + 3 = 15$

$2x = -18$ $\quad$ $2x = 12$

$x = -9$ $\quad$ $x = 6$

9. (9 points) Frodo rode his bicycle to visit his uncle Bilbo who lives 50 kilometers away. On the return trip, Frodo was anxious to get home and his speed was 2 kilometers per hour faster. If the total time for the round trip was 10 hours, what was Frodo's speed on the way to Bilbo's house?

	D	R	T
There	50	X	$\frac{50}{X}$
Back	50	$X+2$	$\frac{50}{X+2}$

Let X = Frodo's speed on the way to Bilbo's house.

$\left(\begin{matrix}\text{Time}\\ \text{There}\end{matrix}\right) + \left(\begin{matrix}\text{Time}\\ \text{Back}\end{matrix}\right) = 10$ hours

$$\frac{50}{X} + \frac{50}{X+2} = 10$$

(Divide by 10.)

$$\frac{5}{X} + \frac{5}{X+2} = 1$$

$LCD = X(X+2)$

$$X(X+2)\left[\frac{5}{X} + \frac{5}{X+2}\right] = X(X+2)$$

$$5(X+2) + 5X = X^2 + 2X$$

$$5X + 10 + 5X = X^2 + 2X$$

$$X^2 - 8X - 10 = 0$$

$$X = \frac{8 \pm \sqrt{64+40}}{2}$$

$$= \frac{8 \pm \sqrt{4 \cdot 26}}{2}$$

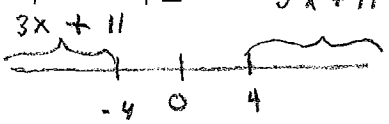
$$= \frac{8 \pm 2\sqrt{26}}{2}$$

The negative solution is extraneous.

$X = 4 + \sqrt{26}$

10. (8 points) Solve the following inequality. Show all work.

$|3x + 11| \geq 4$



$3X + 11 \leq -4$ OR $4 \leq 3X + 11$

$3X \leq -15$ $\quad -7 \leq 3X$

$X \leq -5$ OR $-\frac{7}{3} \leq X$

$(-\infty, -5] \cup [-7/3, \infty)$

11. (8 points) Solve the following inequality. (Hint: try factoring by grouping.) Show all work.

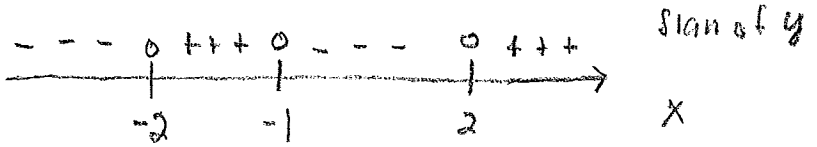
$y = x^3 + x^2 - 4x - 4 \geq 0$

$X^2(X+1) - 4(X+1) \geq 0$

$(X^2 - 4)(X+1) \geq 0$

$(X+2)(X-2)(X+1) \geq 0$

$X = -2, 2, -1$ are the Critical Points.



The solution set of the inequality is

$[-2, -1] \cup [2, \infty)$

$-2 \leq X \leq -1$ OR $2 \leq X$

12.(9 points) Solve the following inequality. Show all work.

$y = \dfrac{2x-5}{x+4} \geq 0$

Zeros of Top

$2x - 5 = 0$

$x = 5/2$

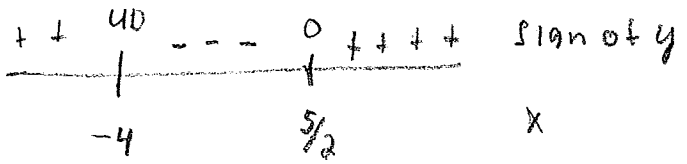
Zeros of Bottom

$x + 4 = 0$

$x = -4$

Critical Numbers

$x = 5/2, -4$



The solution set of the inequality is

$x < -4$ OR $5/2 \leq x$

$(-\infty, -4) \cup [5/2, \infty)$

13.(5 bonus points) Derive the quadratic formula by completing the square on $ax^2 + bx + c = 0$. Show all work.

$ax^2 + bx + c = 0$

$a\left(x^2 + \frac{b}{a}x\right) = -c$

$a\left(x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2\right) = \frac{b^2}{4a} - c$

$a\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a}$

$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$

$\sqrt{\left(x + \frac{b}{2a}\right)^2} = \sqrt{\frac{b^2 - 4ac}{4a^2}}$

$x + \frac{b}{2a} = \pm \frac{\sqrt{b^2 - 4ac}}{2a}$

$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$